Relations and Functions (Mathematics)

Relations

- A **relation** is a set of ordered pairs, usually defined by some sort of rule. It can be plotted onto the number plane.
- The **domain** is the set of all the first elements (**abscissae**) of the ordered pairs (the permitted x values if graphing the relation).
- The **range** is the set of all second elements (**ordinates**) of the ordered pairs (the permitted y values if graphing the relation).

Function

A **function** is a special type of relation, whereby no x-value (abscissae) can be repeated.
All functions are relations but not all relations are functions.

Dependent and Independent Variables

The x-number is called the **independent variable**, and the y-number is called the **dependent variable** because its value depends on the x-value chosen.

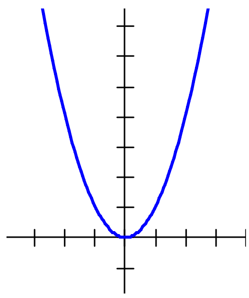
Vertical Line Test

- If a vertical line cuts the graph once only, it is a **function**.
- If a vertical line cuts the graph more than once, it is a **relation**.

Even Functions

Even functions have line symmetry about the y-axis.

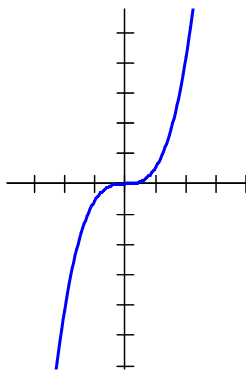
$$\boxed{f(x) = f(-x)}$$



Odd Functions

Odd functions have point symmetry about the origin.

$$\boxed{-f(x) = f(-x)}$$



Graphs of Functions and Relations

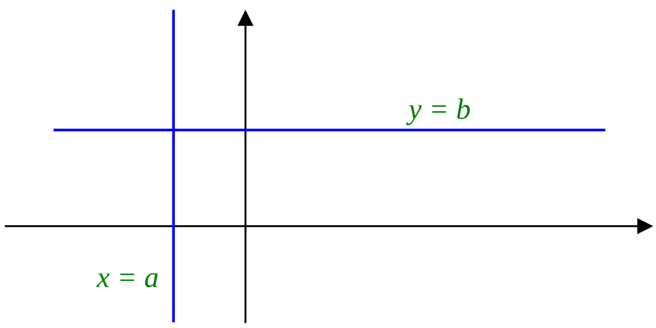
Show important features such as:

- x and y intercepts (if possible)
- turning points
- discontinuous parts of the graph

In addition, find the **domain** and **range**.

Straight Lines of the form $x = a$ go through a on the x-axis; **parallel to y-axis**

Straight Lines of the form $y = b$ go through b on the y-axis; **parallel to x-axis**



Straight lines of the form y = mx + b or ax + by + c = 0

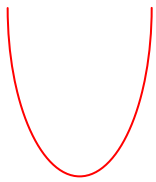
These can be graphed on a number plane by:

- Finding x and y intercepts
- Drawing up a box of values
- Use gradient (Vertical Run / Horizontal Run) and y-intercept (b)

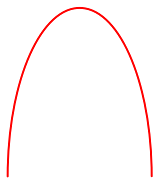
The Parabola (a function)

The graph whose equation is in the form $y = ax + by + c$ where a, b and c are constants, $a \neq 0$ has the shape of a parabola.

If $a > 0$ (leading coefficient greater than zero), then the parabola is **concave up**.



If $a < 0$ (leading coefficient less than zero), then the parabola is **concave down**.



The a values (coefficient of x^2) affect the steepness of the parabola.

Vertex of the Parabola

To find the **axis of symmetry**, use

$$\boxed{x = \frac{-b}{2a}}$$

The axis lies halfway between the two roots.

The Circle (a relation)

The equation of a circle centre (0, 0), radius r units is given by

$$\boxed{x^2 + y^2 = r^2}$$

D: $-r \leq x \leq r$; R: $-r \leq y \leq r$

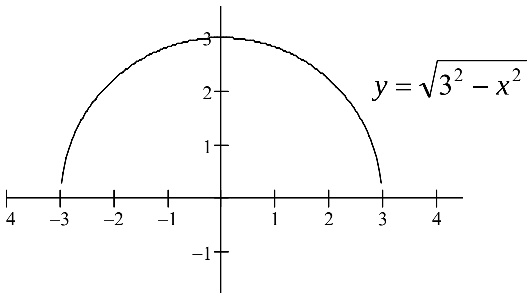
The equation of a circle centre (a, b) radius r units is given by

$$\boxed{(x-a)^2 + (y-b)^2 = r^2}$$

The Semi-circle (a function)

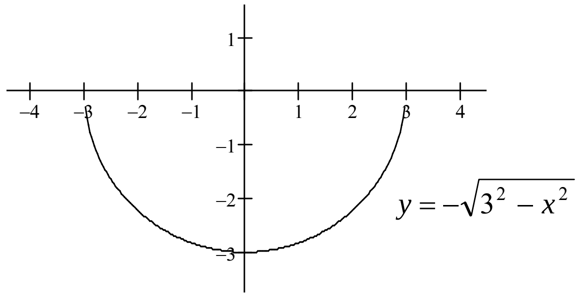
Upper semi-circle (above x-axis)

$$\boxed{y=\sqrt{r^2-x^2}}$$



Lower semi-circle (below x-axis)

$$\boxed{y=-\sqrt{r^2-x^2}}$$



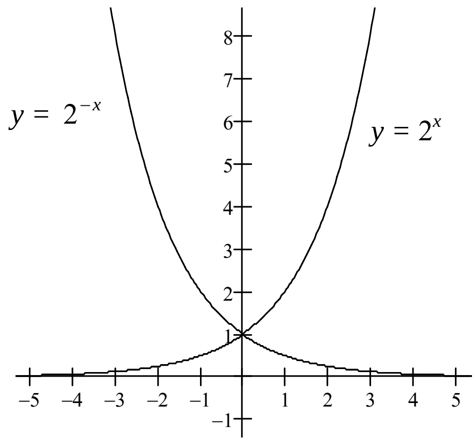
The Exponential Function

This is in the form

$$\boxed{y = a^x}$$

- All (basic) exponential graphs go through (0, 1)
- x-axis is an asymptote

Negative powers reflect the graph in the y-axis.



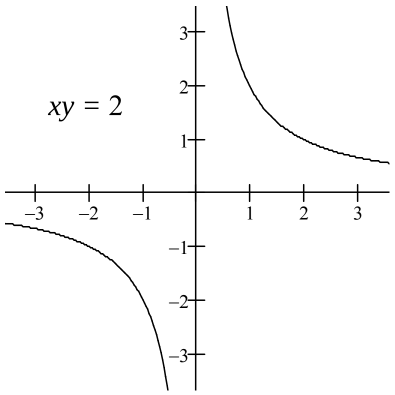
The Hyperbola (rectangular)

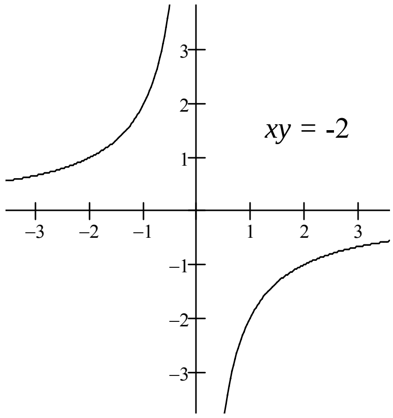
The hyperbola is a function with equation in the form

$$\boxed{y = \frac{k}{x} \text{ or } xy = k}$$

where k is a constant

- The hyperbola consists of two branches in opposite quadrants
- If k is positive, the branches are in the 1st and 3rd quadrants
- If k is negative, the branches are in the 2nd and 4th quadrants
- The x and y axes are asymptotes
- The graph is always discontinuous at the point where the denominator becomes equal to zero
- For $xy = k$, Domain: all real x, $x \neq 0$; Range: all real y, $y \neq 0$





Regions

$$\left.\begin{matrix}\le\\ \ge\end{matrix}\right\}\quad \text{Solid Line}$$

$$\left.\begin{matrix}<\\ >\end{matrix}\right\}\quad \text{Dotted Line}$$

In general, pick a point and use it as a test for the correct region.