JUNIOR INTER (AIEEE & EAMCET)

TRIGONOMETRIC EQUATIONS & INVERSE TRIGONOMETRY

17. $2\tan^{-1} x = \cos^{-1}\dfrac{1-x^2}{1+x^2}, x \geq 0$ $\quad 2\tan^{-1} x = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), |x| < 1$

18 $2\tan^{-1} x = \pi - \sin^{-1} x\left(\dfrac{2x}{1+x^2}\right) = \pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), if x > 1$

19. If $Tan^{-1}x + Tan^{-1}y + Tan^{-1}z = \pi$ then $x + y + z = xyz$

20, If $Tan^{-1}x + Tan^{-1}y + Tan^{-1}z = \dfrac{\pi}{2}$ then $xy + yz + xz = 1$

21. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \dfrac{\pi}{2}$ then $x^2 + y^2 + z^2 + 2xyz = 1$

22.If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ then $x^2 + y^2 + z^2 + 2xyz = 1$

PREVIOUS YEARS EAMCET AND AIEEE PROBLEMS

6. If $x \in (-\pi, \pi)$ such that $y = 1 + |\cos x| + |\cos^2 x| + ...$ and $8^y = 64$ then y = __

(EAMCET 2007)

Hint : $y = 1 + |\cos x| + |\cos x|^2 + ... = \dfrac{1}{1 - |\cos x|} = 2$ (as8^y=64 gives y=2)

$\Rightarrow |\cos x| = \dfrac{1}{2} \Rightarrow x = 60°$ or $120°$

Thus $y = 1 + \dfrac{1}{2} + \dfrac{1}{4} = ... = \dfrac{a}{1-r} = 2$

7. The equation $\sqrt{3}\sin x + \cos x = 4$ has----solutions (EAMCET2001)

Hint: as $|C| \geq \sqrt{a^2 + b^2}$ No solution exists

8: $\tan\left(\dfrac{\pi}{4} + \dfrac{1}{2}Cos^{-1}\dfrac{a}{b}\right) + \tan\left(\dfrac{\pi}{4} - \dfrac{1}{2}Cos^{-1}\dfrac{a}{b}\right) =$

Hint: Put $\frac{1}{2}Cos^{-1}\frac{a}{b}$=x $\Rightarrow \cos 2x = \frac{a}{b}$

$\Rightarrow \tan(\frac{\pi}{4}+x)+\tan(\frac{\pi}{4}-x) = 2\sec 2x = \frac{2}{\cos 2x} = \frac{2b}{a}$

9. Range of $Sin^{-1}x + Cos^{-1}x + Tan^{-1}x$ is

Hint: as

$Sin^{-1}x + Cos^{-1}x = \frac{\pi}{2}$ is valid when

$-1 \prec x \succ 1$

Thus Range of $Sin^{-1}x + Cos^{-1}x + Tan^{-1}x$ is =

$\frac{\pi}{2}+\left[-\frac{\pi}{4},\frac{\pi}{4}\right] = [\pi/4, 3\pi/4]$

10.. If $x^2+y^2+z^2 = r^2$, then $Tan^{-1}\left(\frac{xy}{zr}\right)+Tan^{-1}\left(\frac{yz}{xr}\right)+Tan^{-1}\left(\frac{xz}{yr}\right) =$

Hint:

Put x=y=z=1 $\Rightarrow r^2 = 3 \Rightarrow r = \sqrt{3}$

$\Rightarrow 3\tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{2}$

11 .If $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$ find x

Hint: $\frac{\pi}{2} - \cot^{-1}x + 2\cot^{-1}x = 120° \Rightarrow \cot^{-1}x = 30° \Rightarrow x = \sqrt{3}$

12: If $(Tan^{-1}x)^2 + (Cot^{-1}x)^2 = 5\pi^2/8$, then x =

Hint: Put x=-1

$(Tan^{-1}-1)^2 + (Cot^{-1}-1)^2 = (\frac{-\pi}{4})^2 + (3\frac{\pi}{4})^2 = \frac{5\pi^2}{8}$

13. If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$ find x = ?

Hint : $\sin^{-1}x - \left[\frac{\pi}{2} - \sin^{-1}x\right] = \frac{\pi}{6}$

$\sin^{-1}x - \frac{\pi}{2} + \sin^{-1}x = \frac{\pi}{6} \Rightarrow 2\sin^{-1}x = 120°$

$\sin^{-1} x = 60^\circ \qquad \Rightarrow x = \frac{\sqrt{3}}{2}$

14. $\sum_{i=1}^{n} \cos^{-1} \alpha_i = 0$ then $\sum_{i=1}^{n} \alpha_i =$

Hint : $\cos^{-1} \alpha_1 + \cos^{-1} \alpha_2 ..... = 0$

$\Rightarrow \cos^{-1} \alpha_1 = \cos^{-1} \alpha_2 ..... = 0 \quad \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 ..... = 1$

$\Rightarrow \sum \alpha_i = \sum i = n$

15: $\sin^{-1}(\sin 10) = ...$

$\sin^{-1}(\sin 10) = \sin^{-1} \sin(10x57^0) = \sin^{-1} \sin(570^0)$

$= \sin^{-1} \sin(540 + 30) = \sin^{-1}(-30^0) = \sin^{-1} \sin(3\pi - 10x57) = (3\pi - 10)$

EXTRA QUESTIONS

1. If cot$x$ + cosec$x$ = $\sqrt{3}$ the principal value of $x - \frac{\pi}{6}$ is

   1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{6}$

2. If tan2x = cot3x, then the solutions are in A.P. with common difference

   1) $\frac{\pi}{5}$ 2) $\frac{2\pi}{5}$ 3) $\frac{\pi}{2}$ 4) 2π

3. The equation $\sqrt{3}\sin x + \cos x = 4$ has

   1) Only one solution 2) Two solutions
   3) Infinitely many solutions 4)No solution

4. If $\log_2 \sin x - \log_2 \cos x - \log_2 (1 - \tan^2 x) = -1$ then

   1) $x = \frac{n\pi}{2} + \frac{\pi}{8}$ 2) $x = n\pi - \frac{\pi}{8}$ 3) $\tan 2x = 1$ 4) none

5. If $\frac{1}{6}\sin x, \cos x, \tan x$ are in G.P., then x is equal to

   1) $n\pi \pm \frac{\pi}{3}, n \in Z$ 2) $2n\pi \pm \frac{\pi}{3}, n \in Z$ 3) $n\pi + (-)^n \frac{\pi}{3}, n \in Z$ 4) none of these

6. The value of x which satisfies $8^{1+|\cos x|+|\cos x|^2+\ldots} = 64$ in $[-\pi,\pi]$ is

1) $\pm\frac{\pi}{2},\pm\frac{\pi}{3}$ 2) $\pm\frac{\pi}{3},\pm\frac{2\pi}{3}$ 3) $\pm\frac{\pi}{2},\pm\frac{\pi}{6}$ 4) $\pm\frac{\pi}{6},\pm\frac{\pi}{3}$

7. If α,β are solutions of $a\cos x + b\sin x = c$ then $\cos\alpha+\cos\beta =$

1) $\frac{2ac}{c^2-a^2}$ 2) $\frac{2ac}{a^2+b^2}$ 3) $\frac{c^2-a^2}{b^2+a^2}$ 4) $\frac{b}{a}$

8. If $x\cos\alpha + y\sin\alpha = 2a, xos\beta + y\sin\beta = 2a$ and $2\sin\frac{\alpha}{2}\sin\frac{\beta}{2} = 1,$ then

1) $\cos\alpha+\cos\beta = \frac{2ax}{x^2+y^2}$ 2) $\cos\alpha\cos\beta = \frac{2a^2-y^2}{x^2+y^2}$

3) $y^2 = 4a(a-x)$ 4) $\cos\alpha+\cos\beta = 2\cos\alpha\cos\beta$

9. If α,β are the solutions of $a\cos 2\theta + b\sin 2\theta = c$, then $\tan\alpha\tan\beta =$

1) $\frac{c+a}{c-a}$ 2) $\frac{2b}{c+a}$ 3) $\frac{c-a}{c+a}$ 4) none

10. The smallest positive root of the equation $\tan x - x = 0$ lies in

1) I quadrant 2) II quadrant 3) III quadrant 4) IV quadrant

11. $Tan^{-1}\left(\tan\frac{2\pi}{3}\right) =$

1) $\frac{\pi}{12}$ 2) $\frac{-\pi}{3}$ 3) $\frac{3\pi}{4}$ 4) $\frac{\pi}{6}$

12. $\sin\left(2Tan^{-1}\frac{3}{4}\right) =$

1) $\sin\left(4Tan^{-1}\frac{1}{3}\right)$ 2) $\sin\left(2Tan^{-1}\frac{1}{3}\right)$ 3) $\cos\left(2Tan^{-1}\frac{1}{7}\right)$ 4) $\cos\left(4Tan^{-1}\frac{1}{7}\right)$

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13. If $Tan^{-1}2, Tan^{-1}3$ are two angles of a triangle, then the third angle is

1) 30^0 2) 45^0 3) 60^0 4) 75^0

14. If in $\Delta ABC, C$ is a right angle, then $Tan^{-1}\left(\frac{a}{b+c}\right)+Tan^{-1}\left(\frac{b}{c+a}\right) =$

1) $\pi/6$ 2) $\pi/4$ 3) $\pi/3$ 4) $5\pi/2$

15. $Cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)=$

1) $Sin^{-1}x+Sin^{-1}y$ 2) $Cos^{-1}x+Cos^{-1}y$ 3) $Tan^{-1}x+Tan^{-1}y$ 4) $Cot^{-1}x+Cot^{-1}y$

16. $\tan\left[2Tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)\right]=$

1) x 2) 2x 3) x / 2 4) none

17. $Tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=$

1) $\frac{\pi}{3}-\frac{x}{2}$ 2) $\frac{\pi}{4}-\frac{x}{2}$ 3) $\frac{\pi}{3}+\frac{x}{2}$ 4) $\frac{\pi}{4}+\frac{x}{2}$

18. $2Tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)=$

1) $Cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$ 2) $Cos^{-1}\left(\frac{b+a\cos x}{a-b\cos x}\right)$

3) $Cos^{-1}\left(\frac{b-a\cos x}{a+b\cos x}\right)$ 4) $Cos^{-1}\left(\frac{b-a\cos x}{a-b\cos x}\right)$

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19. $Tan^{-1}\frac{a-b}{1+ab}+Tan^{-1}\frac{b-c}{1+bc}+Tan^{-1}\frac{c-a}{1+ca}=$

1) $\pi/4$ 2) $\pi/2$ 3) 0 4) π

20. If a, b, c have the same sign, then

$Cot^{-1}\left(\frac{ab+1}{a-b}\right)+Cot^{-1}\left(\frac{bc+1}{b-c}\right)+Cot^{-1}\left(\frac{ca+1}{c-a}\right)=$

1) 0 2) $\pi/2$ 3) π 4) 2π

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21. $\sum_{m=1}^{n}Tan^{-1}\frac{1}{2r^2}=\frac{\pi}{4}$

1) $Tan^{-1}(n^2+n+1)$ 2) $Tan^{-1}(n^2-n+1)$ 3) $Tan^{-1}\frac{n^2+n}{n^2+n+2}$ 4) none of these

22. If $\alpha=Cot^{-1}(\sqrt{\cos 2\theta})-Tan^{-1}(\sqrt{\cos 2\theta})$, then $\sin\alpha=$

1) $\sin^2\theta$ 2) $\cos^2\theta$ 3) $\tan^2\theta$ 4) $\cot^2\theta$

23. If $\frac{1}{2} \le x \le 1$ then $Cos^{-1}x + Cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right) =$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $2Cos^{-1}x - \frac{\pi}{3}$

24. If $Cos^{-1}x + Cos^{-1}y + Cos^{-1}z = \pi$, then $x^2 + y^2 + z^2 + 2xyz =$

1) 0 2) 1 3) xyz 4) 2xyz

25. If $Sin^{-1}x + Sin^{-1}y + Sin^{-1}z = \frac{3\pi}{2}$, then the value of

$x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$ is

1) – 1 2) 0 3) 1 4) 3

26. If $Sin^{-1}x + 4Cos^{-1}x = \pi$, then x =

1) $1/2$ 2) $1/\sqrt{2}$ 3) $\sqrt{3}/2$ 4) 1

27. If $3Sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4Cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2Tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$ then x =

1) $\sqrt{2}$ 2) $1/\sqrt{2}$ 3) $\sqrt{3}$ 4) $1/\sqrt{3}$

28. If $Tan^{-1}(x+1) + Tan^{-1}(x-1) = Tan^{-1}(8/31)$, then x =

1) 1 2) 1/2 3) -1/2 4) 1/4

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29.. The trigonometric equation $Sin^{-1}x = 2Sin^{-1}a$, has a solution for

1) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ 2) all real values of a 3) $|a| < \frac{1}{\sqrt{2}}$ 4) $|a| \ge \frac{1}{\sqrt{2}}$

30. Range of $Sin^{-1}x + Cos^{-1}x + Tan^{-1}x$ is

1) $[0, \pi)$ 2) $(0, \pi]$ 3) $[\pi/4, 3\pi/4]$ 4) $[0, \pi]$

31. If cot$x$ + cosec$x$ = $\sqrt{3}$ the principal value of $x-\frac{\pi}{6}$ is

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{6}$

32. If tan2x = cot3x, then the solutions are in A.P. with common difference

1) $\frac{\pi}{5}$ 2) $\frac{2\pi}{5}$ 3) $\frac{\pi}{2}$ 4) 2π

33. The general solution of sin$x$ – 3sin2x + sin3x = cos$x$ – 3cos2x + cos3x is x =

1) $n\pi+\frac{\pi}{8}$ 2) $\frac{n\pi}{2}+\frac{\pi}{8}$

3) $(-1)^n\frac{\pi}{8}+\frac{n\pi}{2}$ 4) $2n\pi+\cos^{-1}\left(\frac{3}{2}\right)$

34. The equation $4\sin^2x + 4\sin x + a^2 - 3 = 0$ posses a solution if

1) $-2 \le a \le 2$ 2) $-1 \le a \le 1$

3) $-3 \le a \le 3$ 4) $-\frac{1}{2} \le a \le \frac{1}{2}$

35. If α, β are two different values of θ lying between 0 and 2π which satisfy the equation $6\cos\theta + 8\sin\theta = 9$ then the value of $\sin(\alpha + \beta)$ =

1) $-\frac{24}{25}$ 2) $\frac{9}{25}$ 3) $\frac{24}{25}$ 4) $-\frac{9}{25}$

36. If $1+\sin x+\sin^2x+\sin^3x+\ldots..+\infty = 4+2\sqrt{3}$; $0 < x < \pi$ then

1) $x=\frac{\pi}{6}$ 2) $x=\frac{\pi}{3}$

3) $x=\frac{\pi}{3}$ or $\frac{\pi}{6}$ 4) $x=\frac{\pi}{3}$ or $\frac{2\pi}{3}$

37. Sin[$\cos^{-1}(3/5) + \tan^{-1}(2)$] =

1) $\frac{2}{5\sqrt{5}}$ 2) $-\frac{2}{5\sqrt{5}}$ 3) $\frac{3}{5\sqrt{5}}$ 4) $\frac{-3}{5\sqrt{5}}$

38. $\sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{63}{16}\right)=$

1) 0 2) $\frac{\pi}{2}$ 3) π 4) 2π

39. If $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{2\pi}{3}$ then x =

1) $\sqrt{3}$ 2) $\frac{1}{\sqrt{3}}$ 3) $2+\sqrt{3}$ 4) $2-\sqrt{3}$

40. $\sec^{-1}\left(\sqrt{1+x^2}\right)+\operatorname{cosec}^{-1}\frac{\sqrt{1+y^2}}{y}+\cot^{-1}\left(\frac{1}{z}\right)=3\pi$ then

1) $x + y + z = 0$ 2) $x + y + z = 1$
3) $x + y + z = xyz$ 4) None

KEY FOR 1 TO 40 PROBLEMS

1) 4 2) 1 3) 4 4) 3 5) 2 6) 2 7) 2 8) 3

9) 3 10) 3 11) 2 12) 3 13) 2 14) 2 15) 2 16) 1

17) 2 18) 1 19) 3 20) 3 21) 3 22) 3 23) 3 24) 2

25) 2 26) 3 27) 4 28) 4 29) 3 30) 3 31)4 32)1

33) 2 34)1 35)3 36)4 37)2 38)3 39)4 40)3