1.1 Electric Charge

- Of almost more than 100 fundamental particles of matter, three most important are electron, proton and neutron. Their masses are $m_e = 9.1 \times 10^{-31}$ kg, $m_p \cong m_n = 1.6 \times 10^{-27}$ kg respectively.

- Gravitational force of attraction between two electrons 1 cm apart is 5.5×10^{-67} N, whereas electrical force of repulsion due to electric charge on them is 2.3×10^{-24} N which is much stronger.

- Electric charge can be positive or negative. Traditionally, charge of proton is considered positive and that of electron negative although reverse sign convention would have made no difference.

- Like charges repel each other and unlike charges attract. Electroscope is used to detect charges.

- Electrons revolving around the nucleus are weakly bound as compared to the force with which protons are bound inside the nucleus. Hence, during exchange of electrons between two bodies, electrons get transferred from one body to the other.

- SI unit of charge is coulomb denoted by C. It is the charge passing in 1 second through any cross-section of a conductor carrying 1 ampere current. Magnitude of charge on an electron or a proton is 1.6×10^{-19} C.

- Electric charge, like mass, is a fundamental property which is difficult to define.

1.2 Quantization of Electric Charge

The magnitudes of all charges found in nature are in integral multiple of a fundamental charge ($Q = ne$). This fact is known as the quantization of charges. This fundamental charge is the charge of an electron and is denoted by e.

All fundamental charged particles possess charge having magnitude e. For example, a proton and a positron (positive electron) possess positive charge (+e). Atom as a whole is electrically neutral as there are equal number of protons and electrons in it. This fact has been verified with an accuracy of 1 in 10^{20}.

No theory has been able to satisfactorily explain the quantization of charges so far.

Protons and neutrons are believed to be made up of more fundamental particles called quarks. Quarks are of two types; 'up quark' possessing +(2/3)e charge and 'down quark' possessing -(1/3)e charge. The independent existence of quarks is not detected so far.

1.3 Conservation of Electric Charge

Irrespective of any process taking place, the algebraic sum of electric charges in an electrically isolated system always remains constant. This statement is called the law of conservation of charge.

In an electrically isolated system, a charge can neither enter nor leave it. Any charge-less matter or radiation can enter or leave the system. γ- ray photon entering the system may produce an electron-positron pair which as a whole being electrically neutral does not alter the original charge of the system.

1.4 Charging by Induction

If two identical spheres, one carrying electric charge Q and the other no charge, are brought in contact and separated, each will possess equal charge Q / 2 after separation. Thus an uncharged sphere gets charged. Another method of charging a substance is explained as under.

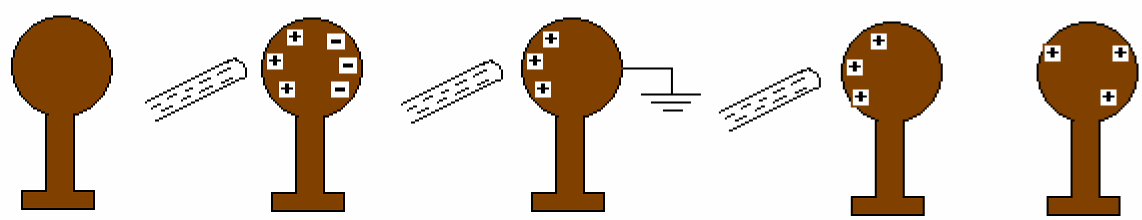


Fig. 1 Fig. 2 Fig. 3 Fig. 4 Fig. 5

Fig. 1 shows a sphere with zero charge.
Fig. 2 shows a plastic rod rubbed with fur which acquires negative charge brought close to the sphere. This repels free electrons on the sphere to a part away from the rod leaving part of the sphere closer to the rod positively charged.
Fig. 3 shows the electrons on the sphere conducted to the earth by earthing the sphere.
Fig. 4 shows that the positive charge is still retained by the sphere even on removal of the earthing.
Fig. 5 shows electrons redistributed on the sphere so that the positive charge is spread all over the surface of the sphere.

This shows that a body can be charged without bringing in physical contact with another charged substance. This phenomenon is called induction of electric charge.

1.5 Coulomb's Law

"The electrical force (Coulombian force, F) between two stationary point charges (q_1 and q_2) is directly proportional to the product of the charges (q_1q_2) and inversely proportional to the square of the distance (r^2) between them." This statement is known as Coulomb's law.

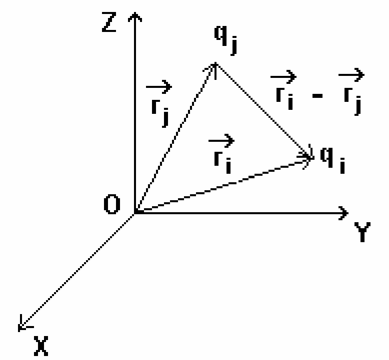
$$F \propto \frac{q_1q_2}{r^2} \quad \Rightarrow \quad F = k\frac{q_1q_2}{r^2} \quad \Rightarrow \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

where F is in N, q_1 and q_2 are in C, r is in m and $k = 9 \times 10^9\ Nm^2C^{-2}$ in vacuum is the proportionality constant. $\varepsilon_0 = 8.9 \times 10^{-12}\ C^2N^{-1}m^{-2}$ is the electrical permittivity in vacuum.

If the charges are in medium other than vacuum, then the electrical permittivity of the medium, ε, should be used in the above equation in place of ε_0. The ratio $\varepsilon / \varepsilon_0$ is called relative permittivity, ε_r, of that medium. The Coulomb's law for any medium is written as

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}, \text{ where } \varepsilon = \varepsilon_0\,\varepsilon_r$$

Coulomb's Law in Vector Form



Let q_i and q_j be two electrical like charges (both positive or both negative) having position vectors $\vec{r}_i$ and $\vec{r}_j$ respectively in a Cartesian co-ordinate system. The force, $\vec{F}_{ij}$, acting on charge q_i due to q_j, directed from q_j to q_i, is given by

$$\vec{F}_{ij} = k\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|^2}\frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|} = k\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|^3}(\vec{r}_i - \vec{r}_j)$$

where $\dfrac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|}$ is the unit vector in the direction from q_j to q_i.

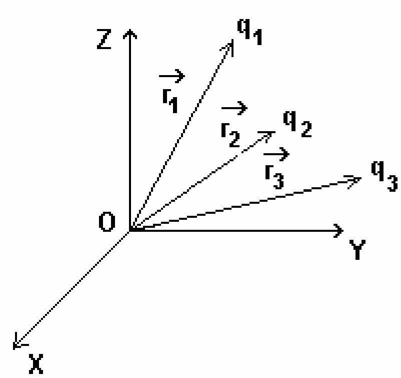
Similarly, the force, $\vec{F}_{ji}$, acting on charge q_j due to q_i, directed from q_i to q_j, is given by

$$\vec{F}_{ji} = k\frac{q_i q_j}{|\vec{r}_j - \vec{r}_i|^2}\frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|} = k\frac{q_i q_j}{|\vec{r}_j - \vec{r}_i|^3}(\vec{r}_j - \vec{r}_i)$$

where $\dfrac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|}$ is the unit vector in the direction from q_i to q_j. Note that $\vec{F}_{ij} = -\vec{F}_{ji}$.

1.6 Forces between More than Two Charges: The Superposition principle

" When more than one Coulombian force are acting on a charge, the resultant Coulombian force acting on it is equal to the vector sum of the individual forces."



Consider charges q_1, q_2 and q_3 having position vectors $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ respectively. Let $\vec{F}_{21}$ and $\vec{F}_{23}$ be the forces acting on charge q_2 due to charges q_1 and q_3 respectively.

Then, $\vec{F}_{21} = k\dfrac{q_2 q_1}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$ and

$$\vec{F}_{23} = k\frac{q_2 q_3}{|\vec{r}_2 - \vec{r}_3|^3}(\vec{r}_2 - \vec{r}_3)$$

and from the principle of superposition, the resultant force acting on charge q_2 is

$$\vec{F}_2 = k \frac{q_2 q_1}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) + k \frac{q_2 q_3}{|\vec{r}_2 - \vec{r}_3|^3}(\vec{r}_2 - \vec{r}_3)$$ and in short,

$$\vec{F}_2 = kq_2 \sum_{\substack{j=1 \\ j \neq 2}}^{3} \frac{q_j}{|\vec{r}_2 - \vec{r}_j|^3}(\vec{r}_2 - \vec{r}_j)$$

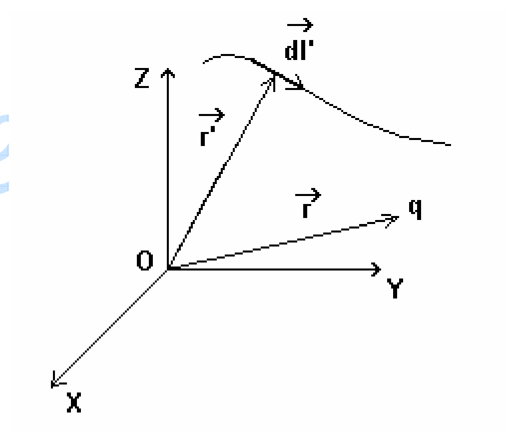
In general, the force acting on charge q_i due to a system of N electric charges will be

$$\vec{F}_i = kq_i \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{q_j}{|\vec{r}_i - \vec{r}_j|^3}(\vec{r}_i - \vec{r}_j)$$

1.7 Continuous Distribution of Charges

The continuous distribution of charges can be of three types:
(1) Line Distribution, (2) Surface Distribution and (3) Volume Distribution

Line Distribution



Let $\vec{r'}$ = position vector of a point on the curved line as shown in the figure,

$\lambda(\vec{r'})$ = linear charge density at the above point,

$\vec{dl'}$ = length of a small line element at that point,

then the amount of charge in that line element

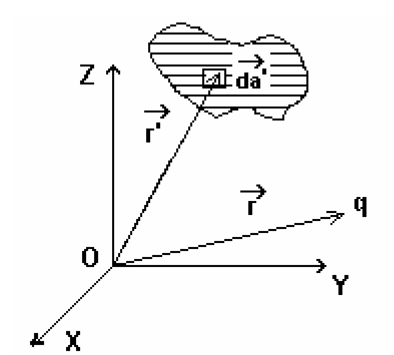
$= \lambda(\vec{r'})\,|\vec{dl'}|$ and

the electrical force acting on charge q having position vector $\vec{r}$ is given by

$$\vec{dF} = \frac{kq\,\lambda(\vec{r'})\,|\vec{dl'}|}{|\vec{r} - \vec{r'}|^3}(\vec{r} - \vec{r'})$$

On integrating, total force $$\vec{F} = kq \int_l \frac{\lambda(\vec{r'})\,|\vec{dl'}|}{|\vec{r} - \vec{r'}|^3}(\vec{r} - \vec{r'})$$

Surface Distribution

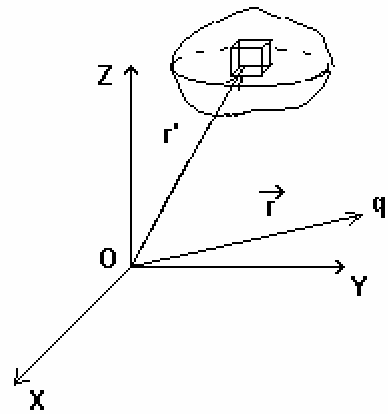


Let $\sigma(\vec{r'})$ = surface charge density at a point having position vector, $\vec{r'}$, on any surface,

$\vec{da'}$ = area vector of a small area around that point as shown in the figure, On calculating the force acting on any charge q having position vector, $\vec{r}$, due to the charge in the small surface element, $\vec{da'}$, and integrating over the entire surface we get total force

$$\vec{F} = kq \int_a \frac{\sigma(\vec{r'})|\vec{da'}|}{|\vec{r} - \vec{r'}|^3} (\vec{r} - \vec{r'})$$

Volume Distribution



Let $\rho(\vec{r'})$ = volume charge density at a point having position vector, $\vec{r'}$, in any volume,

$\vec{dV'}$ = small volume element of the entire volume,

On calculating the force acting on any charge q having position vector, $\vec{r}$, due to the charge in the small volume element, $\vec{dV'}$, and integrating over the entire volume we get total force

$$\vec{F} = kq \int_V \frac{\rho(\vec{r'})|\vec{dV'}|}{|\vec{r} - \vec{r'}|^3} (\vec{r} - \vec{r'})$$

1.8 Electric Field

The region around a system of charges in which the effect of electric charge is prevailing is called the electric field of that particular system of electric charges.

" The force acting on a unit positive charge at a given point in an electric field of a point charge or of a system of charges is called the electric field (or the intensity of electric field) $\vec{E}$ at that point."

Thus, $$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q} = k \sum_{j=1}^{N} \frac{q_j}{|\vec{r} - \vec{r_j}|^3} (\vec{r} - \vec{r_j})$$

Here, q_1, q_2, ..., q_N are the sources of the electric field.

The unit of electric field intensity in SI system is $N\,C^{-1}$ (or $V\,m^{-1}$).

Noteworthy points for an electric field

1) The electric charge used to measure electric field intensity is called a test charge.

2) If we know electric field intensity at all the points in the electric field, there is no need to know the source charges or their locations in the field.

3) The test charge should be as small as possible to ensure that its presence makes no change in the original field.

4) The direction of force experienced by a positive charge at any point is the direction of electric field at that point.

5) Faraday first introduced the concept of an electric field which is a physical reality.

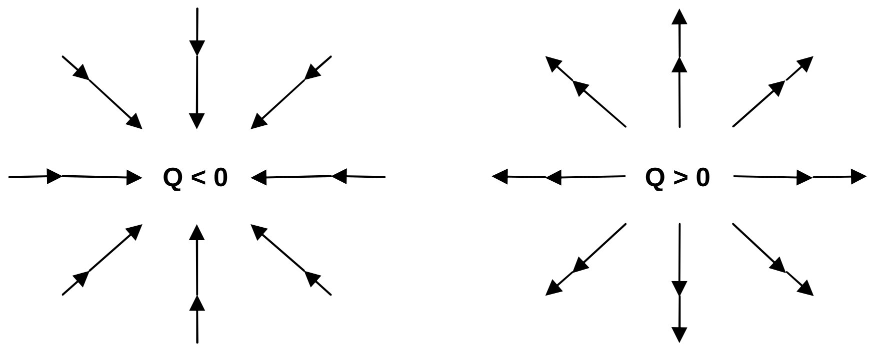
1.8 (a) Electric Field due to a Point Charge

Taking the position of charge Q as origin, the electric force due to it acting on charge q at a distance r from it will be

$$\vec{F} = \frac{kQq}{r^2}\hat{r}$$

Hence, electric field intensity due to charge Q will be,

$$\vec{E} = \frac{\vec{F}}{q} = \frac{kQ}{r^2}\hat{r}$$



The figure shows the electric field due to point charges in two dimensions. Actual field spreads radially in all directions intersecting spherical surface perpendicularly, centre of the sphere located at the point charge, and is directed outwards if the charge is positive and inwards if the charge is negative.

The strength of the electric intensity decreases away from the charge as indicated by decreasing length of arrows.

The electric field due to more than one charge is equal to the vector sum of the individual electric fields due to all the charges.

1.9 Electric Dipole

A system of two equal and opposite charges, separated by a finite distance, is called an electric dipole. If the charges are q and -q and 2a is the distance between them, electric dipole moment of the dipole is

$$\vec{p} = q(2\vec{a})$$

Electric dipole moment is a vector quantity and its direction is from the negative electric charge to the positive electric charge. Its unit is coulomb-meter (Cm).

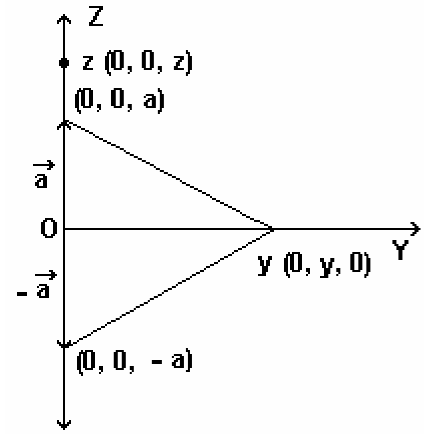
The total charge on an electric dipole is zero, but its electric field is not zero, since the position of the two opposite charges is different.

1.9 (a) Electric Field of a Dipole

To find the electric field of a dipole, let origin of the co-ordinate system be at its mid-point. Let the +q charge be on positive Z-axis and -q charge be on negative Z-axis and the separation between them be 2a.

The position vector of +q charge is $\vec{r_1}$ (0, 0, a) and that of -q charge is $\vec{r_2}$ (0, 0, -a). The electric field due to this dipole at any point having position vector $\vec{r}$ is given by

$$\vec{E}(\vec{r}) = k\left[\frac{(+q)(\vec{r} - \vec{r_1})}{|\vec{r} - \vec{r_1}|^3} + \frac{(-q)(\vec{r} - \vec{r_2})}{|\vec{r} - \vec{r_2}|^3}\right]$$



For a point z on z-axis

having position vector, $\vec{r}$ = (0, 0, z),

$$\vec{E}(z) = k\left[\frac{q\,(0,0,z-a)}{|(z-a)|^3} - \frac{q\,(0,0,z+a)}{|(z+a)|^3}\right]$$

$$= k\left[\frac{q}{(z-a)^2} + \frac{-q}{(z+a)^2}\right]\hat{p}$$

$$= \frac{kq(4za)}{(z^2 - a^2)^2}\hat{p}$$

But 2aq = p,

$$\therefore \quad \vec{E}(z) = \frac{2kpz}{(z^2 - a^2)^2}\hat{p} = \frac{2kp}{z^3}\hat{p}$$ (ignoring a^2 compared to z^2 if z >> a)

For a point y on y-axis having position vector, $\vec{r}$ = (0, y, 0),

$$\vec{E}(y) = k\left[\frac{q(0,y,-a)}{(y^2 + a^2)^{\frac{3}{2}}} - \frac{q(0,y,a)}{(y^2 + a^2)^{\frac{3}{2}}}\right]$$

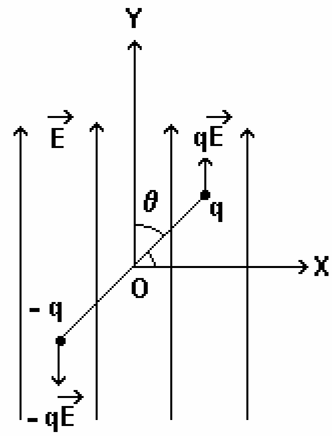
$$= \frac{kq}{(y^2 + a^2)^{\frac{3}{2}}}(0,0,-2a) = -\frac{kq(2a)}{(y^2 + a^2)^{\frac{3}{2}}}\hat{p}$$

$$= -\frac{kp}{(y^2 + a^2)^{\frac{3}{2}}}\hat{p}$$

$$= -\frac{kp}{y^3}\hat{p}$$ (if y >> a)

1.10 The Behaviour of an Electric Dipole in a Uniform Electric Field

An electric dipole $\vec{p} = q(2\vec{a})$ is kept in a uniform electric field $\vec{E}$ making an angle θ with it. The origin of the co-ordinate system, O, is at the centre of the dipole and $\vec{E}$ is directed along the positive Y-axis.



The resultant of qE and -qE forces acting on +q and -q charges respectively being zero, the dipole is in translational equilibrium. But as the two forces have different lines of action, the dipole will experience a torque.

The torques acting on charge +q due to force qE and on charge -q due to force -qE respectively with respect to origin are

$$\vec{\tau}_1 = \vec{a} \times q\vec{E} \quad \text{and} \quad \vec{\tau}_2 = (-\vec{a}) \times (-q\vec{E}) = \vec{a} \times q\vec{E}$$

The total torque acting on the dipole is

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = (\vec{a} \times q\vec{E}) + (\vec{a} \times q\vec{E}) = 2q\,\vec{a} \times \vec{E}$$

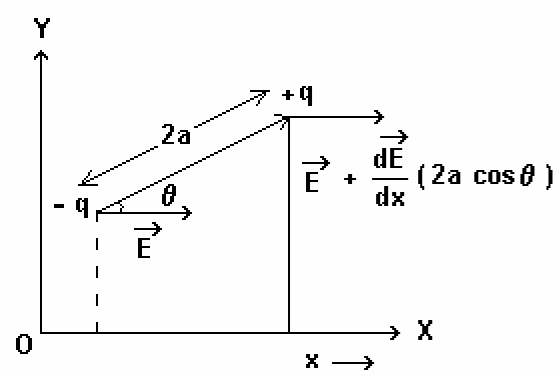
$$\therefore \quad \vec{\tau} = \vec{p} \times \vec{E} \quad \text{(in anti-clockwise direction)}$$

The magnitude of this torque is $\tau = pE\sin\theta$ and its direction is perpendicularly coming out of the plane of figure.

The dipole rotates due to this torque till the angle θ reduces to zero and the dipole aligns itself along the direction of the electric field. This is the equilibrium position of the dipole (about which dipole oscillates in absence of damping) and if it has to be rotated by some angle from this position, work will have to be done equal to the change in potential energy of the dipole.

1.11 Behaviour of an Electric Dipole in a Non-uniform Electric Field

In a non-uniform electric field, the intensity of the field being different at different points, different forces act on the two charges of the dipole. Hence the dipole experiences a linear displacement in addition to rotation.



Let the electric field intensity be $\vec{E}$ at -q charge and let it increase linearly in the X-direction. Let the x-coordinate of -q charge be x. Then from the figure, x-coordinate of +q charge is $x + 2a\cos\theta$. Also the electric intensity near +q charge will be $E + \frac{dE}{dx} 2a\cos\theta$. The electrical force acting on these charges will be $-q\vec{E}$ and $+q\left(\vec{E} + \frac{d\vec{E}}{dx} 2a\cos\theta\right)$.

The net force on the dipole being $q \frac{d\vec{E}}{dx} 2a\cos\theta$, the dipole will have acceleration in the positive x-direction in addition to rotation in the clockwise direction. The rotation will stop when the dipole aligns in the direction of the field (assuming damping is present) but the translation will continue in the positive x-direction.

When a dry comb charged by rubbing with dry hair is brought close to small pieces of paper, electric dipole is induced in them in the direction of non-uniform electric field. This exerts a net force on the pieces of paper which get attracted to the comb.

1.12 Electric Field Lines

Michael Faraday introduced the concept of electric field lines and called them lines of force. An electric field line is a curve drawn in the electric field in such a way that the tangent to the curve at any point is in the direction of electric intensity at that point.

1.12 (a) Characteristics of Electric Field Lines

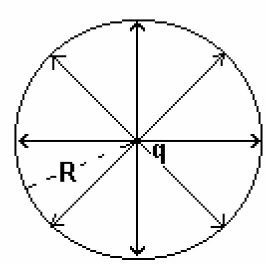
(1) The tangent drawn at any point on the electric field line indicates the direction of electric intensity at that point.

(2) Two electric field lines do not intersect because if they do then two tangents can be drawn at their point of intersection which is not possible.

(3) The distribution of electric field lines in the region of the electric field gives the intensity of electric field in that region.

The number of electric field lines passing perpendicularly through unit cross-sectional area about a point is proportional to the electric intensity at that point. Hence, the field lines will be crowded where the electric intensity is more and sparse where it is less.

Let there be N (an arbitrary number) number of field lines perpendicular to the surface of a sphere of radius R due to a point charge q as shown in the figure. This is not the flux.



Now, the number of field lines per unit area is proportional to the electric intensity.

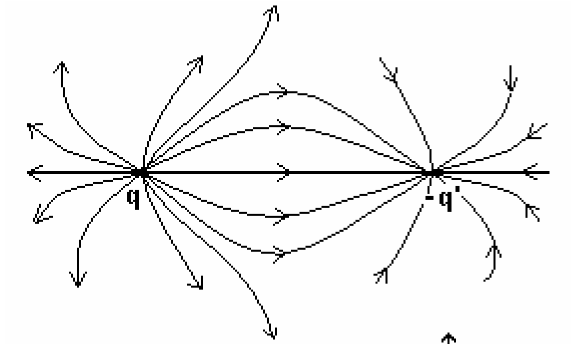
$$\therefore \quad \frac{N}{4\pi R^2} \propto \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \qquad \therefore \quad N = \frac{\beta q}{\varepsilon_0}$$

where, β is the proportionality constant value of which can be determined from the initially assigned arbitrary number, N.

In the case of an electric dipole, the number of field lines originating from +q charge enter into -q charge as both the charges are of the same magnitude. But if one charge is q and the other is -q', where q > q', then the number of electric field lines leaving the charge +q will be

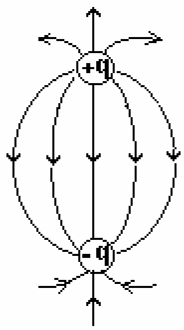
$N = \dfrac{\beta q}{\varepsilon_0}$ and the number of electric field lines entering -q' charge will be

$N' = \dfrac{\beta q'}{\varepsilon_0}$.



Thus out of N number of lines N' number of lines enter the charge -q' and the remaining lines become radial at large distances and move to infinity as shown in the figure.

The electric field lines are used for geometrical representation of electric field and are imaginary. The electric field is a reality.



(4) The field lines of a uniform electric field are mutually parallel and equidistant.

(5) The field lines of a stationary electric charge do not form close loops.

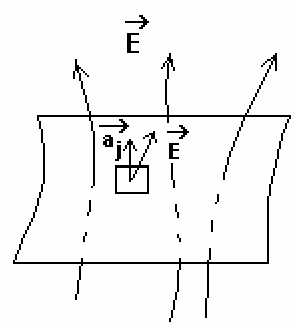
The adjoining figure shows electric field lines of an electric dipole.

1.13 Electric Flux

Consider an arbitrary surface in an electric field as shown in the figure. An infinitely small element of it can be considered flat if its surface is not highly irregular. It can be considered as a vector quantity having magnitude equal to its area and direction normal to its surface.

If $\Delta \vec{a}_j$ = area vector of jth element and

$\vec{E}_j$ = electric field at jth element (which can be considered constant as the area vector is very small),



then electric flux associated with jth element = $\vec{E}_j \cdot \Delta \vec{a}_j$

and the total flux linked with the entire surface is

$$\phi = \lim_{|\Delta \vec{a}_j| \to 0} \sum_j \vec{E}_j \cdot \Delta \vec{a}_j = \int_{\text{surface}} \vec{E} \cdot d\vec{a}$$

1.14 Gauss's Theorem (or Law)

"The total electric flux associated with any closed surface is equal to the ratio of the total electric charge enclosed by the surface to ε_0."

$$\therefore \quad \phi = \oint \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} \sum q$$

(Note: ε_0 is to be used if the medium in closed surface is vacuum or atmospheric air, else the permittivity, ε, of the medium has to be used.)

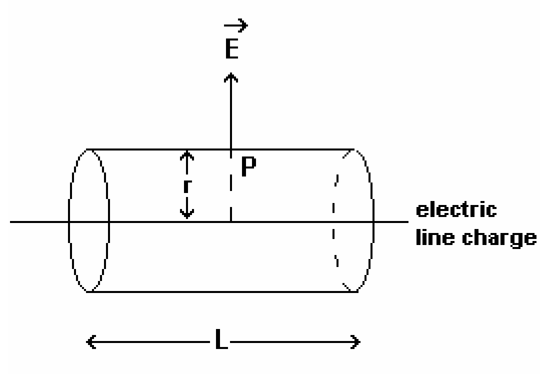
The electric field $\vec{E}$ in the above equation is the resultant electric field due to all charges whether inside or outside the enclosed surface, but the summation of charges on the RHS of the above equation is the algebraic sum of the charges enclosed by the surface.

1.15 Application of Gauss's Theorem

(i) Electric Field due to an Infinitely Long Straight Charged Wire or Line Charge

Let λ = uniform charge density along the length of the conductor.

From symmetry, the magnitude of electric field at all points like P over the curved surface of the cylinder of radius r and length L, whose axis coincides with the conductor, will be the same. The direction of the field at all points on this surface and also at all points on two ends of the cylinder is radially outwards if $\lambda > 0$.



Using Gauss's Law,

$$\int \vec{E} \cdot \vec{da} = \frac{q}{\varepsilon_0} \qquad \therefore \quad 2\pi r L E = \frac{\lambda L}{\varepsilon_0}$$

(Surface integration for all points on the two ends of the cylinder will be zero as the field lines are perpendicular to area vector.)

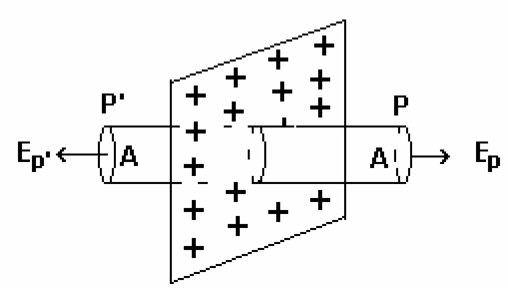
$$\therefore \quad E = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r} \qquad \therefore \quad \vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r}\,\hat{r} = \frac{2k\lambda}{r}\,\hat{r}$$

(ii) Electric Field due to a Uniformly Charged Infinite Plane Sheet or Sheet of Charge

Let σ = uniform surface charge density on an infinite plane sheet.

P and P' are points at a perpendicular distance r on either sides of the charged plane. By symmetry, electric intensity at P and P' will have equal magnitude and opposite direction. If the charge on the plane is positive/negative, the direction of the electric intensity will be away/towards the plane. Consider a closed cylinder with equal lengths on either side of the plane, from P to P'. As the electric intensity is perpendicular to the plane, the flux linked with the curved surface of the cylinder is zero. As the points P and P' are equidistant from the charged plane, the magnitude of electric intensity are the same.

$\therefore \quad E_P = E_{P'} = E$ and $E_PA + E_{P'}A = 2EA$ is the total flux coming out of the cross-sectional area, A, of the cylinder. The closed cylindrical surface encloses the charge $q = \sigma A$.



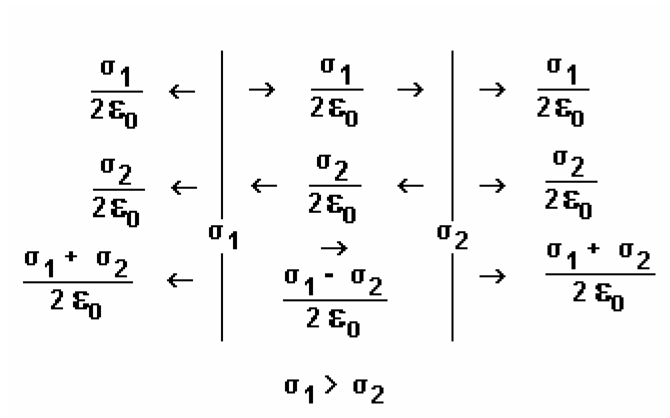
Using Gauss's Law,

$$\int \vec{E} \cdot \vec{da} = \frac{q}{\varepsilon_0} \qquad \therefore \quad 2EA = \frac{\sigma A}{\varepsilon_0} \quad \text{and}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

The above equation shows that electric intensity at any point is independent of its distance from the plane.

If two uniformly charged infinite plates, having surface charge density σ_1 and σ_2, are kept parallel to each other, then the magnitudes and directions of electric intensity at points between and on either sides of planes will be as shown in the figure.

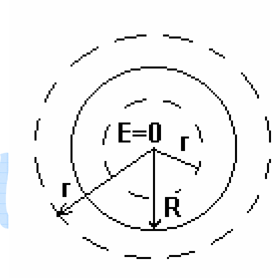


If $\sigma_1 = -\sigma_2 = \sigma$, then the electric intensity between the plates will be σ / ε_0 and on either sides of the two plates will be zero.

(iii) Electric Field due to a Uniformly Charged Thin Spherical Shell

Let σ = uniform surface charge density on a spherical shell of radius R.

(a) For points inside the shell:



Since the charge enclosed in a spherical surface of radius r < R is zero, the electric intensity is zero at all points inside it.

(b) For points outside the shell:

Applying Gauss's Theorem to a spherical surface of radius r > R,

$$4\pi r^2 E(r) = \frac{4\pi R^2 \sigma}{\varepsilon_0}$$

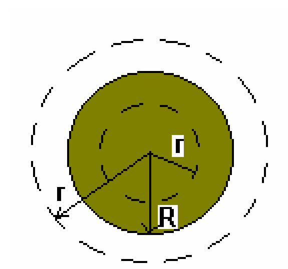
$$\therefore \quad E(r) = \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2} = \frac{q}{4\pi R^2 \varepsilon_0}\frac{R^2}{r^2} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2},$$

where q is the total charge on the spherical shell. Thus for points outside the spherical shell, the entire charge of the spherical shell can be treated as concentrated at its centre.

(iv) Electric Field due to a Uniformly Charged Sphere

Let ρ = uniform volume charge density on a sphere of radius R.

(a) For points inside the sphere:



Applying Gauss's Theorem to a sphere of radius r $\leq$ R,

$$4\pi r^2 E(r) = \frac{4\pi r^3}{3}\frac{\rho}{\varepsilon_0} \qquad \therefore \quad E(r) = \frac{\rho r}{3\varepsilon_0} = E(R)\frac{r}{R}$$

The direction of the field is radially outwards if $\rho > 0$ and inwards if $\rho < 0$.

(b) For points outside the sphere:

Applying Gauss's Theorem to a sphere of radius r, concentric with charged sphere of radius R (r > R),

$$4\pi r^2 E(r) = \frac{4\pi R^3}{3}\frac{\rho}{\varepsilon_0} = \frac{Q}{\varepsilon_0},$$ where Q is the charge on the sphere.

$$\therefore \quad E(r) = \frac{R^3 \rho}{3 r^2 \varepsilon_0} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2} \quad (r > R)$$

Thus, for points outside the sphere, the entire charge of the sphere can be treated as concentrated at its centre.